R2: Solution. Reinforced Brickwork Beam

The effective depth will be based upon the limiting span to depth ratios given in BS EN 1996-1-1, the coordinating dimensions of brickwork coursing and if critical the shear strength of the beam section.
In order to suit the supporting piers width make beam width, b = 440 mm.
Use bricks as soldier courses for aesthetic reasons and so that brickwork in compression is loaded normal to its bed face. Which is compatible with their test procedure and the declared compressive strength of the brick masonry units.
(It should be noted that if brickwork is laid horizontally such that it is loaded in compression across its stretcher face, f_k and f_d must be determined in relation to the actual direction of structural loading).
Trial section is as shown in diagram. As arrangement of section and concrete infilling will allow shear links to be used shear is unlikely to control the effective depth, d.
Overall beam depth = 553mm
Assume Exposure situation MX3 with C35/45 infilling concrete to inner void and actual cover provided to reinforcement is 20mm minimum
As Table NA.9 of the UK National Annex requires a minimum cover of 30mm for this exposure situation austenitic stainless steel reinforcing bars and shear links will be used throughout. 20mm minimum cover required to achieve bond and practical cover requirements.
Effective depth, d = (553-103-20-8-10) = 412mm (d = 410mm)

Now actual effective span, l_{ef} = (3800 + 410)x10^{-3} = 4,21m assuming in this case that d is less than the supporting piers width ($t_1/2 + t_2/2$).
Span/effective depth ratio must not exceed 20 from Table 5.2, therefore max. allowable effective span, l_{ef} = 20 x 410 x 10^{-3} = 8,20m for the serviceability conditions of deflection and cracking
This is greater than the actual effective span of 4,21m.
And l_r limiting = $60b_c$ or $250b_c^2/d$ whichever is the lesser from Eqns. 5.13 and 5.14 respectively, therefore:
$60b_c$ = 60 x 0,440 = 26,4 m > 4,21m therefore satisfactory, and
$250b_c^2/d$ = 250 x $0,440^2$ / 0,410 = 118,05m > 4,21m therefore also satisfactory.

Consider the design load = $1,35G_k + 1,5Q_k$
where 1,35 and 1,5 are the partial safety factors for the unfavourable effects of permanent and variable loads in accordance with BS EN 1990 and the UK National Annex.
Design Load = 1,35 x 20.0 + 1,5 x 8,0
Design Load = 39,00kN/m run

The simply supported design moment is:
M_{Ed} = 39,00 x $4,21^2$ /8 = 86,40kN.m

The design shear force is:
V_{Ed} = 39,00 x 4,21 /2 = 82,10kN

Section bending capacity

The design bending moment, M_{Ed}, is 86,40kN.m

The declared compressive strength of the brick masonry units is 55N/mm^2
Thus normalised compressive strength of the bricks, f_b, is given by:
f_b = 1,0 x 0,85 x 55N/mm^2
f_b = 46,75N/mm^2
where test moisture conditioning factor is 1,0 and 0,85 is the shape factor ,*δ*, both taken from BS EN 772 Part 1.

The characteristic compressive strength of the brickwork is given by:
$f_k = K\, f_b^{0,70}\, f_m^{0,30}$ for General Purpose mortar use
where K is taken as 0,5 for Group 1 clay masonry units in accordance with Table NA.4 of the UK National Annex, hence:
$f_k = 0,5 \times 0,8 \times 46,75^{,7} \times 6,0^{0,3}$ (including 0,8 reduction to *k* for the mortar joints in masonry cross section).
f_k = 10,10N/mm^2
And $f_d = f_k/\gamma_M$ = 10,10/2,0 = 5,05N/mm^2 where γ_M is 2,0 for category I units with class 1 execution control from UK National Annex.

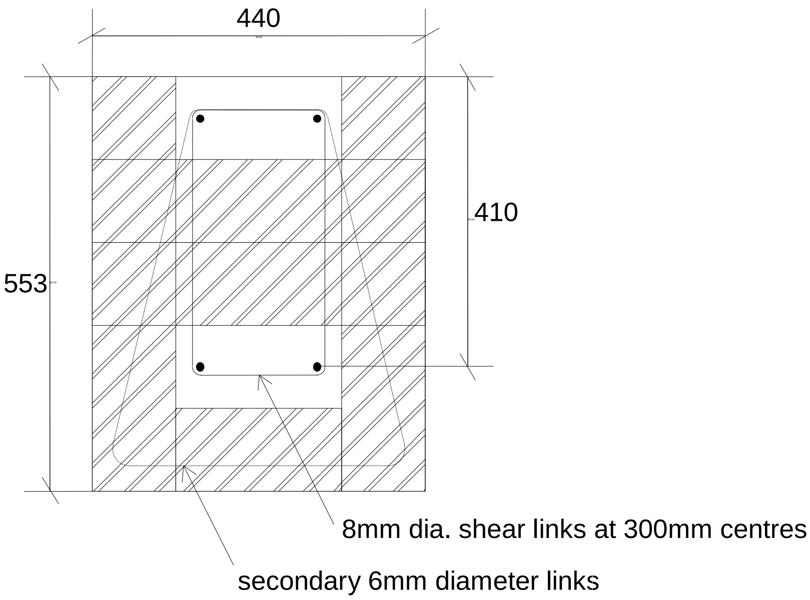

Figure 1: Arrangement of steel in the beam

Design moment of resistance of beam in tension:

Using the *Q* factor approach from BS5628 Part 2 which is compatible with the Eurocode 6 methodology then:
The moment of resistance factor *Q* is given by:
$Q = M_{Ed}/bd^2$ = 86,40 x 10^6 /440 x 410^2 = 1,168

And $Q = 2c(1-c)f_d$
Therefore $1{,}168 = 10{,}10c - 10{,}10c^2$ and solving for c
$c = 0{,}867$ which is the lever arm factor which does not exceed 0,95 limiting
Hence the lever arm z is 0,867 x 410 = 355mm
A_s required = M_{Rd}/f_{yd} z from Eqn (6.23)
Using Grade 500 deformed austenitic stainless steel bars with a yield strength, f_{yk}, of 500N/mm^2 and with γ_M as 1,15 from the UK National Annex
$f_{yd} = 500/1{,}15 = 435$N/mm^2
A_s required = $86{,}40\text{x}\ 10^6$ / 435 x 355
A_s required = 560mm^2

Therefore use 2no. deformed 20mm diameter bars, Austenitic stainless steel, strength Grade 500 (provides 628mm^2).

This area of steel gives a reinforcement percentage of:
628 x 100 / 440 x 410 = 0,35%
which exceeds the minimum value of 0,05% from clause 8.2.3(1)

Limiting compressive design moment of resistance of beam:
$M_{Rd} = 0{,}27 f_d\, bd^2$ from a consideration of a limiting uls compressive stress block of 0,8 x 0,4d in accordance with clause 6.6.2 and the EC6/EC2 methodology.*(It should be noted that EN 1996-1-1 increases this limiting M_{Rd} to 0,4 $f_d\, bd^2$ or 0,3 $f_d\, bd^2$ dependant upon masonry unit type and grouping for the specific design case of cantilever walls)*

$M_{Rd} = 0{,}27 \times 5{,}05 \times 440 \times 410^2 \times 10^{-6}$
M_{Rd} = 100,85kN.m
This exceeds the applied design moment, M_{Ed}, of 86,40kN.m

Section shear capacity

The design shear force, $V_{Ed,}$ is 82,10kN

From Annex J for enhanced shear capacity of section incorporating vertical reinforcement:
Characteristic shear strength of section, $f_{vd} = (0{,}35 + 17{,}5 \times \rho) / \gamma_M$ Eqn. J.1
Where $\rho = A_s/bd$ = (628/440 x 410) = 0,0035
f_{vd} = (0,35 + 17,5 x 0,0035)/2,0 = 0,21N/mm^2
And $f_{vd} \leq 0{,}7/\gamma_M = 0{,}7/2{,}0 = 0{,}35$N/mm^2 limiting
f_{vd} may be increased by the factor $\chi = (2{,}5 - 0{,}25 a_v/d)$
Where a_v = 86,40kN.m/82,10 kN = 1,052m
And a_v/d = 1052/410 = 2,566 < 6 limiting
χ = (2,5 – 0,25 x 2,566) = 1,859
f_{vd} enhanced = 1,859 x 0,21 N/mm^2 = 0,39N/mm^2
And $f_{vd} \leq 1{,}75 / \gamma_M = 1{,}75/2{,}0 = 0{,}88$N/mm^2 limiting
Design shear resistance, $V_{Rd1} = f_{vd}\, b\, d$ Eqn. 6.40
$V_{Rd1} = (0{,}39 \times 440 \times 410 \times 10^{-3}) = 70{,}36$kN
This is less than the applied design shear, $V_{Ed,}$ of 82,10kN and designed shear reinforcement is needed.

With designed shear reinforcement included:
$V_{Ed} \leq V_{Rd1} + V_{Rd2}$
where $V_{Rd1} + V_{Rd2}$ is the total design shear resistance of the beam section
V_{Rd2} is given by Eqn. 6.43 as:

$$V_{Rd2} = 0,9d\frac{A_{SW}}{s}f_{yd}\,(1 + cot\alpha)sin\alpha$$

where α is angle of inclination of the shear reinforcement to the horizontal plane which is 90° here for vertically aligned shear links

and f_{yd} = 200/1,15 = 174N/mm^2 for Grade 200 plain stainless steel bars

Therefore V_{Rd2} = (0,9 x 410 x A_{sw} x 174/300) x 1,0 x 10^{-3} = 0,214A_{sw} kN
where A_{sw} is the cross sectional area of the steel reinforcement required at 300mm horizontal centres which is the maximum spacing allowed as the lesser of 300 mm or 0,75d (308 mm) from clause 8.2.7(6)

Hence 70,36 + 0,214A_{sw} = 82,10kN
and A_{sw} = 55mm^2
Check that the minimum area 0,05% shear steel is provided from clause 8.2.3(5)
Minimum area = 0.0005 x 440 x 410 = 90mm^2

Use 8mm diameter plain bar Austenitic stainless steel, strength Grade 200 closed beam links at 300mm horizontal centres within concrete infill (provides 101mm^2)

Check limiting condition:
$V_{Rd1} + V_{Rd2} \leq 0,25\, f_d\, bd$ Eqn. 6.43
where 0,25 $f_d\, bd$ = 0,25 x 5,05 x 440 x 410 x 10^{-3} = 227,76 kN
This is greater than, V_{Ed}, = 82,10kN and therefore adequate.

Secondary nominal links of 6mm diameter plain bar Austenitic stainless steel will also be required to tie into brickwork section and therefore link spacing of 300mm horizontally suits brickwork cross-joint spacing (75mm brick coursing multiples).

The anchorage bond of the tensile steel has not been considered in this example.